# Logical Relations Amongst Sentences

We've already seen that the truth conditions for certain English sentences are systematically related to each other. Representing the meanings of English sentences in terms of their propositional logic translations allows us to precisely characterize these truth-conditional relationships.

## Entailment

For our purposes, the most important such relation is entailment. We've already seen numerous examples in which one English sentence entails another:

(1) Jim didn't get a raise, and Dwight didn't get a raise. ~p & ~s

(2) Jim didn't get a raise. ~p

Whenever (1) is true, (2) must also be true. In other words, (1) entails (2). The truth tables for the propositional logic translations of (1) and (2) show this relationship clearly:

| p | s | **~p** | ~s | **~p & ~s** |
|---|---|---|---|---|
| T | T | **F** | F | **F** |
| T | F | **F** | T | **F** |
| F | T | **T** | F | **F** |
| F | F | **T** | T | **T** |

- A **entails** B: whenever A is true, B is also true

If A entails B, then there are no rows where A is true but B is false.

Note that (2) does not entail (1), since it is possible for (2) to be true while (1) is false. This relationship also emerges clearly from the above truth table. If A does not entail B, then there will always be a row where A is true and B is false.

## Logical Equivalence

A closely related notion is that of logical equivalence:

(3) It's not true that Jim or Dwight got a raise. ~(p v s)

(4) Jim didn't get a raise, and Dwight didn't get a raise. ~p & ~s

Whenever (3) is true, (4) must also be true. Likewise, whenever (4) is true, (3) must also be true. In other words, (3) and (4) are logically equivalent, since they are true in exactly the same circumstances (they have the same truth conditions).

| p | s | p v s | **~ (p v s)** | ~p | ~s | **~p & ~s** |
|---|---|---|---|---|---|---|
| T | T | T | **F** | F | F | **F** |
| T | F | T | **F** | F | T | **F** |
| F | T | T | **F** | T | F | **F** |
| F | F | F | **T** | T | T | **T** |

- A and B are **log. equivalent**: A and B always have the same truth value

If A and B are logically equivalent, then there are no rows where A and B differ in their truth values.

As the discussion of (3) and (4) suggests, logical equivalence can be understood as mutual entailment. If A and B are logically equivalent, then A entails B, and B also entails A.

## Logical Incompatibility (or "Logical Contrariety", see Löbner)

(5) Jim and Dwight both got raises. p & s

(6) Dwight didn't get a raise. ~s

Clearly, if (5) is true, then (6) is false, and if (6) is true, then (5) is false. In other words, (5) and (6) are logically incompatible, since they cannot both be true.

| p | s | **p & s** | **~s** |
|---|---|---|---|
| T | T | **T** | **F** |
| T | F | **F** | **T** |
| F | T | **F** | **F** |
| F | F | **F** | **T** |

- A and B are **logically incompatible**: A and B cannot both be true

If A and B are logically incompatible, then there are no rows where A and B are both true.

Note that it is possible for (5) and (6) to both be false—imagine that Jim didn't get a raise, but Dwight did. If A and B are logically incompatible, there may still be rows where A and B are both false.

Logical Compatibility

Finally, we have the notion of logical compatibility:

(7) Jim or Dwight got a raise. $p \vee s$

(8) Jim didn't get a raise. $\sim p$

It is possible for (7) and (8) to both be true—again, imagine that Jim didn't get a raise, but Dwight did. This means that (7) and (8) are logically compatible.

| p | s | **p v s** | **~p** |
|---|---|---|---|
| T | T | **T** | **F** |
| T | F | **T** | **F** |
| F | T | **T** | **T** |
| F | F | **F** | **T** |

- A and B are **logically compatible**:
  A and B can both be true

If A and B are logically compatible, then there is a row where A and B are both true.

Logical compatibility and incompatibility are mutually exclusive, as their definitions make clear. If A and B are logically compatible, then they are not logically incompatible, and vice versa.