The Syntax and Semantics of Restricted Quantifiers

The Syntax of Restricted Quantifiers: If *A* and *B* are formulas, then so is $[Qx_n: A]\ B$, where Q is a quantifier symbol and x_n is a variable.

$[Qx_n: A]\ B$

- Q = quantifier symbol, x_n = ind. variable
- *A* = formula (containing the variable x_n)
- *B* = formula (containing the variable x_n)

A is the **restriction** on Q: restricts the quantifier to range over only those individuals that satisfy *A*

B is the **nuclear scope** of Q

restricted quantifier: only ranges over members of *Val*(STUDENT), not all of the individuals in *D*

(1) Most student(s) left. $[Mx_1: \text{STUDENT}(x_1)]\ \text{LEAVE}(x_1)$
Every $\forall x_1$
Some $\exists x_1$

(2) [**Every** happy student] left.
Q restrictor nuc. scope

$[\forall x_1: \text{HAPPY}(x_1)\ \&\ \text{STUDENT}(x_1)]\ \text{LEAVE}(x_1)$

(3) [**At least three** students who are sick] completed the exam.
Q restrictor nuc. scope

$[\geq 3x_1: \text{STUDENT}(x_1)\ \&\ \text{SICK}(x_1)]\ \text{COMPLETE}(x_1, e)$

(4) Peter met [**exactly two** students from Tennessee].
nuc. scope Q restrictor

$[=2x_1: \text{STUDENT}(x_1)\ \&\ \text{FROM}(x_1, t)]\ \text{MEET}(p, x_1)$

(5) [**No** student who failed Semantics 1] passed the midterm and the final.
Q restrictor nuc. scope

no = not+some $\sim[\exists x_1: \text{STUDENT}(x_1)\ \&\ \text{FAIL}(x_1, s)]\ \text{PASS}(x_1, m)\ \&\ \text{PASS}(x_1, f)$

or

no = every+not $[\forall x_1: \text{STUDENT}(x_1)\ \&\ \text{FAIL}(x_1, s)]\ \sim(\text{PASS}(x_1, m)\ \&\ \text{PASS}(x_1, f))$

or

no = exactly zero $[=\emptyset x_1: \text{STUDENT}(x_1)\ \&\ \text{FAIL}(x_1, s)]\ \text{PASS}(x_1, m)\ \&\ \text{PASS}(x_1, f)$

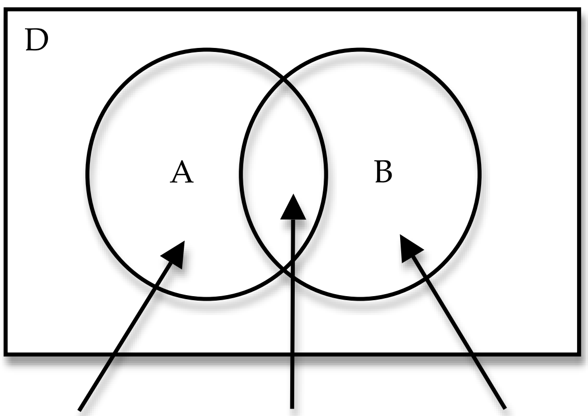
The Semantics of Restricted Quantifiers: The formula [Qx_n: *A*] *B* is true if the relation expressed by Q holds between the set contributed by *A* and the set contributed by *B*. Otherwise, [Qx_n: *A*] *B* is false.

[Mx_1: STUDENT(x_1)] LEAVE(x_1)

Q expresses a relation between A and B by placing requirements on A, B, A – B, A ∩ B, and/or B – A (see below)

the **restriction** on Q: contributes a set of individuals A (the set of students, i.e., *Val*(STUDENT))

the **nuclear scope** of Q: contributes another set of individuals B (the set of individuals who left, i.e., *Val*(LEAVE))



A – B: set of individuals that belong to A, but not B

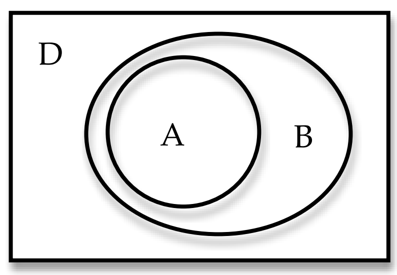
A ∩ B: set of individuals that belong to both A and B

B – A: set of individuals that belong to B, but not A

For any set S, the **cardinality** of S (written |**S**|) is the number of individuals that are members of S.

If every member of A is also a member of B, then A is a **subset** of B (written **A ⊆ B**).

The subset relationship:



The **cardinal quantifiers** impose a requirement on the cardinality of A ∩ B (i.e., the number of individuals that belong to both A and B):

=2 (*exactly two*)	$\lvert A \cap B \rvert = 2$
≥3 (*at least three*)	$\lvert A \cap B \rvert \geq 3$
∃ (*some, a(n), at least one*)	$\lvert A \cap B \rvert \geq 1$
=Ø (*exactly zero, no*)	$\lvert A \cap B \rvert = 0$

The **proportional quantifiers** impose a requirement on the proportional relationship between A and B:

∀ (*every, each, all*)	$A \subseteq B$ or equivalently, $\lvert A - B \rvert = 0$
M (*most*)	$\lvert A \cap B \rvert > \lvert A - B \rvert$ or equivalently, $\frac{\lvert A \cap B \rvert}{\lvert A \rvert} > .5$