Semantic Relations Amongst Sentences

We've already seen that the truth conditions for certain English sentences are systematically related to each other. Representing the meanings of English sentences in terms of their propositional logic translations allows us to precisely characterize these truth-conditional relationships, and to quickly identify them.

Below, we will assume the following basic translations of English sentences into propositional variables:

p = *Jim got a raise.*
s = *Dwight got a raise.*

Entailment

For our purposes, the most important semantic relation is entailment. We've already seen numerous examples in which one English sentence entails another:

(1) Jim didn't get a raise, and Dwight didn't get a raise.
~p & ~s

(2) Jim didn't get a raise.
~p

Whenever (1) is true, (2) must also be true. In other words, (1) entails (2). The truth tables for the propositional logic translations of (1) and (2) show this relationship clearly:

p	s	**~p**	~s	**~p & ~s**
T	T	**F**	F	**F**
T	F	**F**	T	**F**
F	T	**T**	F	**F**
F	F	**T**	T	**T**

If A entails B, then there are no rows where A is true but B is false.

Note that (2) does not entail (1), since it is possible for (2) to be true while (1) is false. This relationship also emerges clearly from the above truth table. If A does not entail B, then there will always be a row where A is true and B is false.

Logical Equivalence

A closely related notion is that of logical equivalence:

(1) Jim didn't get a raise, and Dwight didn't get a raise.
~p & ~s

(3) It's not true that Jim or Dwight got a raise.
~(p v s)

Whenever (3) is true, (1) must also be true. Likewise, whenever (1) is true, (3) must also be true. In other words, (1) and (3) are logically equivalent, since they are true in exactly the same circumstances (they have the same truth conditions).

p	s	~p	~s	**~p & ~s**	(p v s)	**~(p v s)**
T	T	F	F	**F**	T	**F**
T	F	F	T	**F**	T	**F**
F	T	T	F	**F**	T	**F**
F	F	T	T	**T**	F	**T**

If A and B are logically equivalent, then there are no rows where A and B differ in their truth values.

As the discussion of (1) and (3) suggests, logical equivalence can be understood as mutual entailment. If A and B are logically equivalent, then A entails B, and B also entails A.

Logical Denial

Opposed to the notion of logical equivalence is that of logical denial:

(4) Jim or Dwight got a raise.
(p v s)

(4) has exactly the opposite truth conditions from (3), since the two sentences always have different truth values. This makes (4) the logical denial of (3).

p	s	~p	~s	**~p & ~s**	**(p v s)**	**~(p v s)**
T	T	F	F	**F**	**T**	**F**
T	F	F	T	**F**	**T**	**F**
F	T	T	F	**F**	**T**	**F**
F	F	T	T	**T**	**F**	**T**

If A and B are each other's logical denials, then there are no rows where they have the same truth value.

Note that since (1) and (3) possess the same truth conditions, it follows immediately that (4) is also the logical denial of (1).

Logical Incompatibility

Consider next the relationship between (1) and (5):

(1) Jim didn't get a raise, and Dwight didn't get a raise.
~p & ~s

(5) Dwight got a raise.
s

Clearly, if (5) is true, then (1) is false, and if (1) is true, then (5) is false. In other words, (1) and (5) are logically incompatible, since they cannot both be true.

p	**s**	~p	~s	**~p & ~s**	(p v s)	~(p v s)
T	**T**	F	F	**F**	T	F
T	**F**	F	T	**F**	T	F
F	**T**	T	F	**F**	T	F
F	**F**	T	T	**T**	F	T

If A and B are logically incompatible, then there are no rows where A and B are both true.

Note that it is possible for (1) and (5) to both be false—imagine that Jim got a raise, but Dwight didn't. If A and B are logically incompatible, there may still be rows where A and B are both false. (**Question:** how are logical denial and logical incompatibility related?)

Logical Compatibility

Opposed to the notion of logical incompatibility is that of logical compatibility:

(4) Jim or Dwight got a raise.
(p v s)

(6) Dwight didn't get a raise.
~s

It is possible for (4) and (6) to both be true—imagine that Jim got a raise, but Dwight didn't. In other words, (4) and (6) are logically compatible, since they can both be true.

p	s	~p	**~s**	~p & ~s	**(p v s)**	~(p v s)
T	T	F	**F**	F	**T**	F
T	F	F	**T**	F	**T**	F
F	T	T	**F**	F	**T**	F
F	F	T	**T**	T	**F**	T

If A and B are logically compatible, then there is at least one row where A and B are both true.