The Language of Predicate Logic (PredL)

Vocabulary (list of basic expressions)

(i) predicate constants: GREEK, MAN, ... (one-place)
TALLER, FATHER, ... (two-place)
GIVE, BETWEEN, ... (three-place)

(ii) individual constants: a, b, c, d, e, f, ...

(iii) connectives: ~ (negation)
& , ∨ , → (conjunction, disjunction, material implication)

(iv) parentheses: (,)

Syntax (rules for forming grammatical sentences, or "formulas")

(i) If P is an *n*-place predicate constant, and c_1, c_2, ... , c_n are *n* individual constants, then $P(c_1, c_2, \ldots, c_n)$ is a formula of PredL.

(ii) If *A* is a formula of PredL, then so is ~*A*.

(iii) If *A* and *B* are formulas of PredL, then so are (*A* & *B*), (*A* ∨ *B*), and (*A* → *B*).

(iv) Nothing else is a formula.

(Note: typically, we omit the outermost pair of parentheses in a PredL formula. But <u>all</u> other parentheses are necessary to avoid any potential ambiguity.)

Semantics (rules that assign truth conditions to PredL formulas)

Two-step procedure for assigning truth conditions to PredL formulas:

(A) Provide denotations for individual/predicate constants by defining a **model**.

A **model** *M* consists of:

(i) a set *D* of individuals, and

(ii) an "assignment function" *Val*, which assigns a denotation (= semantic value) to each individual/predicate constant in PredL.

(The members of *D* are the inhabitants of our "world", while *Val* serves to establish a relationship between the "words" of PredL and our "world".)

(B) Show how the truth conditions of a PredL formula depend upon the denotations of the individual/predicate constants that appear within it.

(i) If P is a one-place predicate constant and c is an individual constant, then P(c) is true if $Val(c) \in Val(P)$.
(I.e., P(c) is true if the individual denoted by c is a member of the set denoted by P—remember that '∈' stands for 'is a member of'.)
Otherwise, P(c) is false.

Intuition: a one-place predicate "checks" whether an individual possesses a certain property.

(ii) If P is a two-place predicate constant and c_1, c_2 are individual constants, then $P(c_1, c_2)$ is true if $\langle Val(c_1), Val(c_2)\rangle \in Val(P)$.
Otherwise, $P(c_1, c_2)$ is false.

(iii) If P is a three-place predicate constant and c_1, c_2, c_3 are individual constants, then $P(c_1, c_2, c_3)$ is true if $\langle Val(c_1), Val(c_2), Val(c_3)\rangle \in Val(P)$.
Otherwise, $P(c_1, c_2, c_3)$ is false.

Intuition: a two- or three-place predicate "checks" whether two or three individuals stand in a certain relationship to each other.

(iv) The truth conditions for complex formulas constructed with ~, &, ∨, and → are given by our familiar truth tables:

A	**~A**
T	**F**
F	**T**

A	B	**(A & B)**	**(A ∨ B)**	**(A → B)**
T	T	**T**	**T**	**T**
T	F	**F**	**T**	**F**
F	T	**F**	**T**	**T**
F	F	**F**	**F**	**T**