The Syntax and Semantics of Restricted Quantifiers

The Syntax of Restricted Quantifiers: If $A$ and $B$ are formulas, then so is $[Qx_n: A] B$, where $Q$ is a quantifier symbol and $x_n$ is a variable.

- $Q =$ quantifier symbol, $x_n =$ ind. variable
- $A =$ formula (containing the variable $x_n$)
- $B =$ formula (containing the variable $x_n$)

$A$ is the restriction on $Q$; $B$ is the nuclear scope of $Q$, restricts the quantifier to range over only those individuals that satisfy $A$

restricted quantifier: only ranges over members of $Val(STUDENT)$, not all of the individuals in $D$

(1) Most student(s) left.  
   Every  
   Some

   $$[Mx_1: STUDENT(x_1)] \ LEAVE(x_1)$$
   $$\forall x_1 \ \exists x_1$$

(2) Every happy student left.

   $Q$ restrictor nuc. scope

   $$[\forall x_1: HAPPY(x_1) \ & \ STUDENT(x_1)] \ LEAVE(x_1)$$

(3) At least three students who are sick completed the exam.

   $Q$ restrictor nuc. scope

   $$[\geq 3x_1: STUDENT(x_1) \ & \ SICK(x_1)] \ COMPLETE(x_1, e)$$

(4) Peter met exactly two students from Tennessee.

   nuc. scope $Q$ restrictor

   $$[=2x_1: STUDENT(x_1) \ & \ FROM(x_1, t)] \ MEET(p, x_1)$$

(5) No student who failed Semantics 1 passed the midterm and the final.

   $Q$ restrictor nuc. scope

   $no = not+some$  
   $\sim[\exists x_1: STUDENT(x_1) \ & \ FAIL(x_1, s)] \ PASS(x_1, m) \ & \ PASS(x_1, f)$ or 
   $no = every+not$  
   $$[\forall x_1: STUDENT(x_1) \ & \ FAIL(x_1, s)] \ \sim(PASS(x_1, m) \ & \ PASS(x_1, f))$$ or 
   $no = exactly zero$  
   $$[=\emptyset x_1: STUDENT(x_1) \ & \ FAIL(x_1, s)] \ PASS(x_1, m) \ & \ PASS(x_1, f)$$
The Semantics of Restricted Quantifiers: The formula \([Q_{x_1}: A] B\) is true if the relation expressed by \(Q\) holds between the set contributed by \(A\) and the set contributed by \(B\). Otherwise, \([Q_{x_1}: A] B\) is false.

\[
[M_{x_1}: \text{STUDENT}(x_1)] \text{ LEAVE}(x_1)
\]

\(Q\) expresses a relation between \(A\) and \(B\) by placing requirements on \(A, B, A - B, A \cap B,\) and/or \(B - A\) (see below)

- the restriction on \(Q\): contributes a set of individuals \(A\) (the set of students, i.e., \(\text{Val(STUDENT)}\))
- the nuclear scope of \(Q\): contributes another set of individuals \(B\) (the set of individuals who left, i.e., \(\text{Val(LEAVE)}\))

For any set \(S\), the cardinality of \(S\) (written \(|S|\)) is the number of individuals that are members of \(S\).

If every member of \(A\) is also a member of \(B\), then \(A\) is a subset of \(B\) (written \(A \subseteq B\)).

The subset relationship:

The cardinal quantifiers impose a requirement on the cardinality of \(A \cap B\) (i.e., the number of individuals that belong to both \(A\) and \(B\)):

- \(= 2\) (exactly two) \(\Rightarrow |A \cap B| = 2\)
- \(\geq 3\) (at least three) \(\Rightarrow |A \cap B| \geq 3\)
- \(\exists\) (some, \(a(n)\), at least one) \(\Rightarrow |A \cap B| \geq 1\)
- \(= \emptyset\) (exactly zero, no) \(\Rightarrow |A \cap B| = 0\)

The proportional quantifiers impose a requirement on the proportional relationship between \(A\) and \(B\):

- \(\forall\) (every, each, all) \(\Rightarrow A \subseteq B\) or equivalently, \(|A - B| = 0\)
- \(M\) (most) \(\Rightarrow |A \cap B| > |A - B|\) or equivalently, \(\frac{|A \cap B|}{|A|} > .5\)